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CHAPTER 1
REAL NUMBERS

EXERCISE 1.1

1. Use Euclid's division algorithm to find the HCF of:
(i) 135 and 225 (ii) 196 and 38220
(iii) 867 and 255

Sol :

(i) **135 and 225**
Start with the larger integer, that is, 225. Apply the division lemma to 225 and 135, we get

$$225 = 135 \times 1 + 90$$

Since the remainder $90 \neq 0$, we apply the division lemma to 135 and 90, we get

$$135 = 90 \times 1 + 45$$

We consider the new divisor 90 and the new remainder 45, and apply the division lemma to get

$$90 = 45 \times 2 + 0$$

The remainder has now become zero, so our procedure stops.
Since the divisor at this stage is 45, the HCF of 225 and 135 is 45.

(ii) **196 and 38220**
Start with the larger integer, that is, 38220. Apply the division lemma to 38220 and 196, we get

$$38220 = 196 \times 195 + 0$$

The remainder is zero, so our procedure stops.
Since the divisor is 196, the HCF of 38220 and 196 is 196.

(iii) **867 and 255**
Start with the larger integer, that is, 867. Apply the division lemma, we get

$$867 = 255 \times 3 + 102$$

Since the remainder $102 \neq 0$, we apply the division lemma to 255 and 102, to get

$$255 = 102 \times 2 + 51$$

We, consider the new divisor 102 and the new remainder 51, and apply the division lemma, we get

$$102 = 51 \times 2 + 0$$

The remainder has now become zero, so our procedure stops.
Since the divisor at this stage is 51, the HCF of 867 and 255 is 51.

PRACTICE :

1. Use Euclid's division algorithm to find the HCF of:
(i) 420 and 130 (ii) 75 and 243
(iii) 240 and 6552
Ans : (i) 10, (ii) 3 (iii) 24

2. Use Euclid's division algorithm to find the HCF of:
(i) 455 and 84 (ii) 960 and 432
(iii) 657 and 963
Ans : (i) 7 (ii) 48 (iii) 9

2. Show that any positive odd integer is of the form $6q+1$, or $6q+3$, or $6q+5$, where q is some integer.

Sol :

Let us consider a positive odd integer as a. On dividing a by 6, let q be the quotient and r be the remainder. Using Euclid's lemma, we have

$a = 6q + r$ where $0 \leq r < 6$ i.e., 0 1, ...6

$a = 6q + 0 = 6q$

or $a = 6q + 1$

or $a = 6q + 2$

or $a = 6q + 3$

or $a = 6q + 4$

or $a = 6q + 5$

But, $a = 6q$, $a = 6q + 2$, $a = 6q + 4$ are even values of a.
$[6q = 2(3q) = 2m_1, 6q + 2 = 2(3q+1) = 2m_2,$
$6q + 4 = 2(3q+2) = 2m_3]$

But a being an odd integer, we have,

$a = 6q + 1$

or $a = 6q + 3$

or $a = 6q + 5$

PRACTICE :

1. Show that any positive odd integer is of the form $8q+1$, or $8q+3$, or $8q+5$ or $8q+7$ where q is some integer.
Ans : Proof

2. Show that any positive integer is of the form $3q$, or $3q+1$, $3q+2$ where q is some integer.
Ans : Proof

3. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Sol :

By applying the Euclid's division lemma, we can find the maximum number of columns in which an army contingent of 616 members can march behind an army band of 32 members in a parade. HCF of 616 and 32 is equal to maximum number of columns in which 616 and 32 members can march.

Since $616 > 32$, we apply the division lemma to 616 and 32, we get

$$616 = 32 \times 19 + 8$$

Since the remainder $8 \neq 0$, we apply the division lemma to 32 and 8, to get

$$32 = 8 \times 4 + 0$$

The remainder has now become zero, so our procedure stops. Since the divisor at this stage is 8, the HCF of 616 and 32 is 8.

Therefore, the maximum number of columns in which an army contingent of 616 members can march behind an army band of 32 members in a parade is 8.

PRACTICE :

1. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

   Ans :

2. There are 120 boys and 114 girls in a class. The principal of school wish to have maximum as of section, each section has to accommodate equal number of boys and girls. What will be the maximum no. of such section ?

   Ans : 6

4. Use Euclid's division lemma to show that the square of any positive integer is either of the form $3m$ or $3m+1$ for some integer m.

[Hint: Let x be any positive integer then it is of the form $3q$, $3q+1$ or $3q+2$. Now square each of these and show that they can be rewritten in the form $3m$ or $3m+1$.]

Sol :

Let a be an arbitrary positive integer.

Then by Euclid's division algorithm, corresponding to the positive integers a and 3 there exist non-negative integers q and r such that

$$a = 3q + r$$

where, $\quad 0 \leq r < 3$

$$a^2 = 9q^2 + 6qr + r^2 \qquad \text{...(i)}$$

Case-I : $r = 0$

$$a^2 = 9q^2 = 3(3q^2) = 3m$$

where, $\quad m = 3q^2 \qquad$ [From (i)]

Case-II $\; r = 1, \quad a^2 = 9q^2 + 6q + 1 = 3(3q^2 + 2q) + 1$

$$= 3m + 1$$

where, $\quad m = 3q^2 + 2q \qquad$ [From (i)]

Case-III $\; r = 1, \quad a^2 = 9q^2 + 6q + 1 = 3(3q^2 + 2q) + 1$

$$= 3m + 1$$

where, $\quad m = (3q^2 + 4q + 1) \qquad$ [From (i)]

Hence, square of any positive integer is either of the form $3m$ or $3m+1$ for some integer m.

PRACTICE :

1. Use Euclid's division lemma to show that the square of any positive integer is either of the form $4m$ or $4m+1$ for some integer m.

   Ans : Proof

2. Use Euclid's division lemma to show that the square of any positive integer can not be of the form $6m+2$ or $6m+5$ for any integer m.

   Ans : Proof

5. Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m+1$ or $9m+8$.

Sol :

Let us consider an arbitrary positive integer a such that it is in the form of $3q$, $(3q+1)$ or $(3q+2)$.

Case 1 : $a = 3q$,

For, $a = 3q$, $\quad a^3 = (3q)^3 = 27q^3 = 9(9q^3)$

$$= 9m \qquad \text{...(1)}$$

Here we have substituted $3q^3 = m$, where m is an integer.

Case 2 : $a = 3q + 1$

For, $a = 3q+1$, $\quad a^3 = (3q+1)^3$

$$= 27q^3 + 27q^2 + 9q + 1$$

$$= 9(3q^3 + 3q^2 + q) + 1$$

$$= 9m + 1 \qquad \text{...(2)}$$

Here we have substituted $(3q^3 + 3q^2 + q) = m$, where m is an integer.

Case 3 : $a = 3q + 2$

For, $a = 3q+2$, $\quad a^3 = (3q+2)^3$

$$= 27q^3 + 54q^2 + 36q + 8$$

$$= 9(3q^3 + 6q^2 + 4q) + 8$$

$$= 9m + 8 \qquad \text{...(3)}$$

Here we have substituted $(3q^3 + 6q^2 + 4q) = m$, where m is an integer.

From (1), (2), (3) we have :

$$a^3 = 9m,\ (9m+1) \text{ or } (9m+8)$$

Thus, cube of any positive integer can be in the form

$9m$, $(9m+1)$ or $(9m+8)$ for some integer m.

PRACTICE :

1. Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m+1$ or $9m+8$.

   Ans : Proof

2. Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m+1$ or $9m+8$.

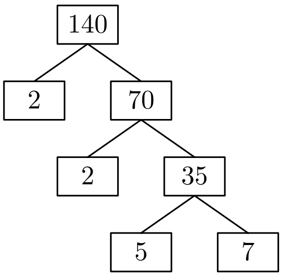
   Ans : Proof

EXERCISE 1.2

1. Express each number as a product of its prime factors:
(i) 140 (ii) 156 (iii) 3825
(iv) 5005 (v) 7429

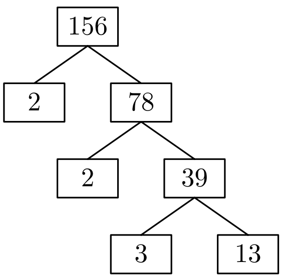
Sol :

(i) 140
Using factor there method we have,



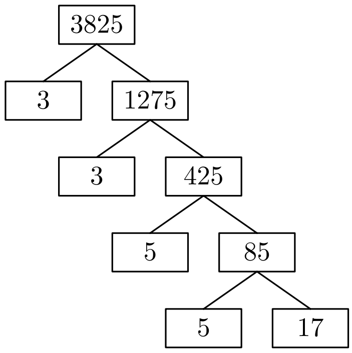
So, $140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$

(ii) 156



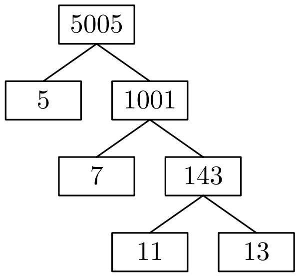
So, $156 = 2 \times 2 \times 3 \times 13$
$= 2^2 \times 3 \times 13$

(iii) 3825



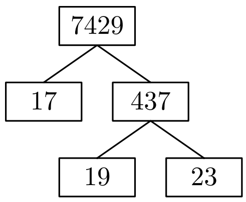
So, $3825 = 3 \times 3 \times 5 \times 5 \times 17$
$= 3^2 \times 5^2 \times 17$

(iv) 5005



So, $5005 = 5 \times 7 \times 11 \times 13$

(v) 7429



So, $7429 = 17 \times 19 \times 23$

PRACTICE :

1. Express each number as a product of its prime factors:
   (i) 88 (ii) 180
   (iii) 2431 (iv) 5200
   (v) 1771

   Ans : (i) $2^3 \times 11$ (ii) $2^2 \times 3^2 \times 5$ (iii) $11 \times 13 \times 17$ (iv) $2^4 \times 5^2 \times 13$ (iv) $7 \times 11 \times 23$

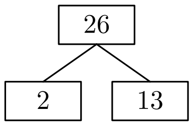
2. Express each number as a product of its prime factors:
   (i) 1575 (ii) 182
   (iii) 3600 (iv) 441
   (v) 20449

   Ans : (i) $3^2 \times 5^2 \times 7$ (ii) $2 \times 7 \times 13$ (iii) $2^4 \times 3^2 \times 5^2$ (iv) $3^2 \times 7^2$ (iv) $11^2 \times 13^2$

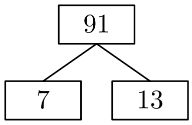
2. Find the LCM and HCF of the following pairs of integers and verify that LCM $\times$ HCF = product of the two numbers.
(i) 26 and 91 (ii) 510 and 92
(iii) 336 and 54

Sol :

(i) 26 and 91



So, $26 = 2 \times 13$



So, $91 = 7 \times 13$

Therefore,

$$\text{LCM }(26,91) = 2 \times 7 \times 13 = 182$$
$$\text{HCF }(26, 91) = 13$$

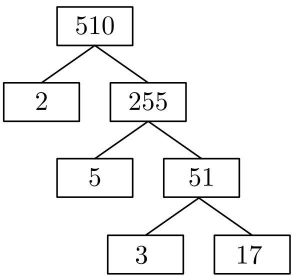
Verification:

$$\text{LCM} \times \text{HCF} = 182 \times 13 = 2366$$

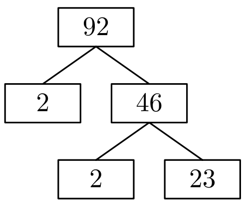
and $26 \times 91 = 2366$

i.e. LCM $\times$ HCF = Product of two numbers.

(ii) 510 and 92



So, $510 = 2 \times 5 \times 3 \times 17$



So, $92 = 2 \times 2 \times 23$

Therefore,

$$\text{LCM }(510,92) = 2 \times 2 \times 5 \times \times 3 \times 17 \times 23$$
$$= 23460$$
$$\text{HCF }(510, 92) = 2$$

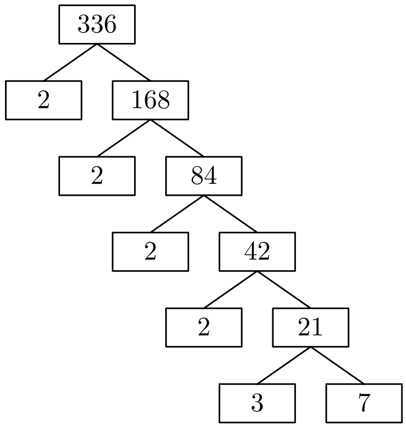
Verification:

$$\text{LCM} \times \text{HCF} = 23460 \times 2 = 46920$$

and $510 \times 92 = 46920$

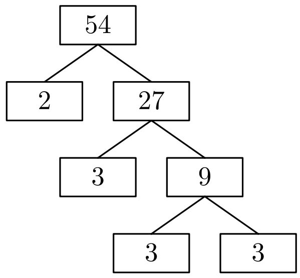
i.e., LCM $\times$ HCF = Product of two numbers

(iii) 336 and 54



So, $336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$

$$= 2^4 \times 3 \times 7$$



So, $54 = 2 \times 3 \times 3 \times 3 = 2 \times 3^3$

Therefore,

$$\text{LCM }(336, 52) = 2^4 \times 3^3 \times 7 = 3024$$
$$\text{HCF }(336, 52) = 2 \times 3 = 6$$

Verification:

$$\text{LCM} \times \text{HCF} = 3024 \times 6 = 18144$$

and $336 \times 54 = 18144$

i.e., LCM $\times$ HCF = Product of two numbers.

PRACTICE :

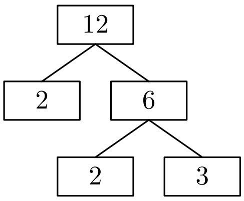
1. Find the LCM and HCF of the following pairs of integers and verify that LCM $\times$ HCF = product of the two numbers.
(i) 90 and 144 (ii) 70 and 50
(iii) 96 and 8

Ans : (i) 720 and 18 (ii) 350 and 10 (iii) 96 and 8

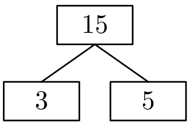
3. Find the LCM and HCF of the following integers by applying the prime factorisation method.
(i) 12, 15 and 21 (ii) 17, 23 and 29
(iii) 8, 9 and 25.

Sol :

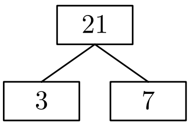
(i) 12, 15 and 21



So, $12 = 2 \times 2 \times 3 = 2^2 \times 3$



So, $15 = 3 \times 5$



So, $21 = 3 \times 7$

Therefore,

$$\text{HCF } (12, 15, 21) = 3$$

$$\text{LCM } (12, 15, 21) = 2^2 \times 3 \times 5 \times 7 = 420$$

(ii) 17, 23 and 29

$$17 = 17$$

$$23 = 23$$

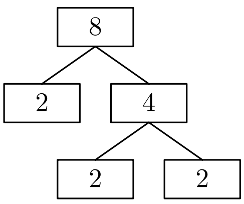
$$29 = 29$$

Therefore,

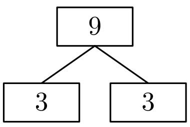
$$\text{HCF } (17, 23, 29) = 1$$

$$\text{LCM } (17, 23, 29) = 17 \times 23 \times 29 = 11339$$

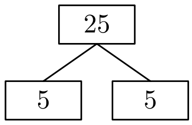
(iii) 8, 9 and 25



So, $8 = 2 \times 2 \times 2 = 2^3$



So, $9 = 3 \times 3 = 3^2$



So, $25 = 5 \times 5 = 5^2$

Therefore,

$$\text{HCF } (8, 9, 25) = 1$$

$$\text{LCM } (8, 9, 25) = 2^3 \times 3^2 \times 5^2 = 1800$$

PRACTICE :

1. Find the LCM and HCF of the following integers by applying the prime factorisation method.
   (i) 24, 15 and 36 (ii) 40, 36 and 126
   (iii) 7, 11 and 13
   Ans : (i) 360 and 3 (ii) 2520 and 2 (iii) 1001 and 1
2. Find the LCM and HCF of the following integers by applying the prime factorisation method.
   (i) 288, 360 and 384 (ii) 6, 72 and 120
   (iii) 20, 12, 16 and 2.
   Ans : (i) 5760 and 24 (ii) 360 and 6 (iii) 240 and 2

4. Given that HCF $(306, 657) = 9$, find LCM $(306, 657)$

Sol :

As we know that,

$$\text{LCM} \times \text{HCF} = \text{Product of numbers}$$

$$\text{LCM} = \frac{306 \times 657}{\text{HCF}}$$

$$= \frac{306 \times 657}{9} = 22338.$$

PRACTICE :

1. Given that HCF $(132, 66) = 33$, find LCM $(132, 66)$
   Ans : 264
2. Given that HCF $(306, 1314) = 18$, find LCM $(306, 1314)$
   Ans : 22338

5. Check whether 6^n can end with the digit 0 for any natural number n.

Sol :

If the number 6^n for any natural number n, end with digit 0, then it would be divisible by 5. That is, the prime factorisation of 6^n would contain the prime 5. This is not possible because $6^n = (2 \times 3)^n = 2^n \times 3^n$ so the only primes in the factorisation of 6^n are 2 and 3 and the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorisation of 6^n.

So, there is no natural number n for which 6^n ends with the digit zero.

PRACTICE :

1. Check whether 7^n can end with the digit 0 for any natural number n.

   Ans : Proof

2. Check whether 8^n can end with the digit 0 for any natural number n.

   Ans : Proof

6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Sol :

(i) $7 \times 11 \times 13 + 13$

$$7 \times 11 \times 13 + 13 = (7 \times 11 + 1) \times 13$$

$$= (77 + 1) \times 13 = 78 \times 13$$

$$= (2 \times 3 \times 13) \times 13$$

$$78 = 2 \times 3 \times 13$$

$$= 2 \times 3 \times 13^2$$

Since, $7 \times 11 \times 13 + 13$ can be expressed as a product of primes, therefore, it is a composite number.

(ii) $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$

$$= (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1) \times 5$$

$$= (1008 + 1) \times 5 = 1009 \times 5$$

$$= 5 \times 1009$$

Since, $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ can be expressed as a product of primes, therefore, it is a composite number.

PRACTICE :

1. Explain why $6 \times 5 \times 3 \times 2 \times 1 + 1$ is not a composite number.

   Ans : Proof

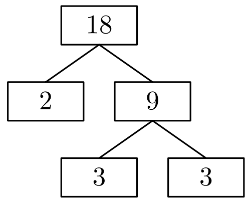
2. Explain why $8 \times 7 \times 6 \times 5 + 5$ is composite numbers.

   Ans : Proof

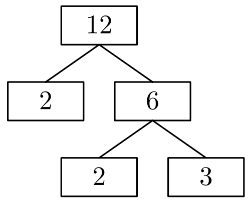
7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Sol :

By taking LCM of time taken (in minutes) by Sonia and Ravi, we can get the actual number of minutes after which they meet again at the starting point after both start as same point and at the same time, and go in the same direction.



$$18 = 2 \times 3 \times 3 = 2 \times 3^2$$



$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$

$$= 2^2 \times 3$$

$$\text{LCM}(18, 12) = 2^2 \times 3^2 = 36$$

Therefore, both Sonia and Ravi will meet again at the starting point after 36 minutes.

PRACTICE :

1. There is a circular path around a sports field. Tania takes 24 minutes to drive one round of the field, while Lavanya takes 18 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

   Ans : 72

2. There is a circular path around a sports field. Tania takes 14 minutes to drive one round of the field, while Lavanya takes 28 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

   Ans : 28 minutes

EXERCISE 1.3

1. Prove that $\sqrt{5}$ is irrational.

Sol :

Let $\sqrt{5}$ be a rational number.

So, we can find co-prime integers a and b $(\neq 0)$ such that

$$\sqrt{5} = \frac{a}{b}$$

$$\sqrt{5}\, b = a$$

Squaring on both sides, we get

$$5b^2 = a^2$$

Therefore, 5 divides a^2

Therefore, 5 divides a

So, we can write

$$a = 5c \text{ for some integer } c.$$

Substituting for a, we get

$$5b^2 = 25c^2$$

$$b^2 = 5c^2$$

This means that 5 divides b^2, and so 5 divides b.

Therefore, a and b have at least 5 as a common factor.

But this contradicts the fact that a and b have no common factor other than 1.

This contradiction arose because of our incorrect assumption that $\sqrt{5}$ is rational.

So, we conclude that $\sqrt{5}$ is irrational.

PRACTICE :

1. Prove that $\sqrt{7}$ is irrational.

Ans : Proof

2. Prove that $\sqrt{10}$ is irrational.

Ans : Proof

2. Prove that $3+2\sqrt{5}$ is irrational.

Sol :

Let, $3+2\sqrt{5}$ is rational number.

That is, we can find co-prime integers a and b $(b \neq 0)$

Such that, $3+2\sqrt{5} = \frac{a}{b}$ where $b \neq 0$

Therefore,
$$\frac{a}{b} - 3 = 2\sqrt{5}$$
$$\frac{a-3b}{b} = 2\sqrt{5}$$
$$\frac{a-3b}{2b} = \sqrt{5}$$
$$\frac{a}{2b} - \frac{3}{2} = \sqrt{5}$$

Since a and b are integers, we get $\frac{a}{2b} - \frac{3}{2}$ is rational, and so $\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $3+2\sqrt{5}$ is rational.

So, we conclude that $3+2\sqrt{5}$ is irrational.

PRACTICE :

1. Prove that $4+3\sqrt{2}$ is irrational.

Ans : Proof

2. Prove that $5+3\sqrt{2}$ is irrational.

Ans : Proof

3. Prove that the following are irrationals.

(i) $\frac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$ (iii) $6+\sqrt{2}$

Sol :

(i) $\frac{1}{\sqrt{2}}$

Let, us assume, to the contrary, that $\frac{1}{\sqrt{2}}$ is rational.

So, we can find co-prime integers a and b $(\neq 0)$ such that

$$\frac{1}{\sqrt{2}} = \frac{a}{b}$$
$$\sqrt{2} = \frac{b}{a}$$

Since, a and b are integers, $\frac{b}{a}$ is rational, and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

So, we conclude that $\frac{1}{\sqrt{2}}$ is irrational.

(ii) $7\sqrt{5}$

Let us assume to the contrary, that $7\sqrt{5}$ is rational.

So, we can find co-prime integers a and b $(\neq 0)$ such that

$$7\sqrt{5} = \frac{a}{b}$$
$$\sqrt{5} = \frac{a}{7b}$$

Since, a and b are integers, $\frac{a}{7b}$ is rational, and so, $\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational.

So, we conclude that $7\sqrt{5}$ is irrational.

(iii) $6+\sqrt{2}$

Let us assume to the contrary, that $\sqrt{2}$ is rational.

Then, $6+\sqrt{2}$ is rational.

So, we can find co-prime integers a and b $(\neq 0)$ such that

$$6+\sqrt{2} = \frac{a}{b}$$
$$6 - \frac{a}{b} = \sqrt{2}$$

Since, a and b are integers, we get $\frac{a}{b}$ is rational and so, $6 - \frac{a}{b}$ is rational and so, $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

So, we conclude that $6+\sqrt{2}$ is irrational.

PRACTICE :

1. Prove that the following are irrationals.

(i) $\frac{1}{\sqrt{3}}$ (ii) $5\sqrt{2}$

(iii) $3+\sqrt{3}$

Ans : Proof

2. Prove that the following are irrationals.

(i) $\frac{1}{\sqrt{5}}$ (ii) $3\sqrt{5}$

(iii) $4+\sqrt{5}$

Ans : Proof

EXERCISE 1.4

1. Without actually performing the long division, state whether the following rational numbers will have *a* terminating decimal expansion or *a* non-terminating repeating decimal expansion.

(i) $\frac{13}{3125}$ (ii) $\frac{17}{8}$ (iii) $\frac{64}{455}$

(iv) $\frac{15}{1600}$ (v) $\frac{29}{343}$ (vi) $\frac{23}{2^3 5^2}$

(vii) $\frac{129}{2^2 5^7 7^5}$ (viii) $\frac{6}{15}$ (ix) $\frac{35}{50}$

(x) $\frac{77}{210}$

Sol :

A rational number has a terminating decimal expansion if and only if the denominator has a prime factorisation of the form $2^n 5^m$, where n and m are non-negative integers.

(i) $\frac{13}{3125}$

Here, $3125 = 5 \times 5 \times 5 \times 5 \times 5$

$$= 5^5 = 2^0 5^5$$

Which is of the form $2^n 5^m$, so the given rational number $\frac{13}{3125}$ will have terminating decimal expansion.

(ii) $\frac{17}{8}$

Here, $8 = 2 \times 2 \times 2 \times 2^3 = 2^3 \cdot 5^0$

Which is of the form $2^n 5^m$, so the given rational number $\frac{17}{8}$ will have terminating decimal expansion.

(iii) $\frac{64}{455}$

Here, $455 = 5 \times 7 \times 13$,

Which is not of the form $2^n 5^m$, so the given rational number $\frac{64}{455}$ will not have terminating decimal expansion. If will have a non-terminating repeating decimal expansion.

(iv) $\frac{15}{1600}$

Here, $1600 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$

$$= 2^6 \times 5^2,$$

Which is of the form $2^n 5^m$, so the given rational number $\frac{15}{1600}$ will have terminating decimal expansion.

(v) $\frac{29}{343}$

Here, $343 = 7 \times 7 \times 7 = 7^3$,

Which is not of the form $2^n 5^m$, so the given rational number $\frac{29}{343}$ will not have terminating decimal expansion. It will have a non-terminating repeating decimal expansion.

(vi) $\frac{23}{2^3 5^2}$

Here, denominator $2^3 5^2$ is of the form $2^n 5^m$, so the given rational number $\frac{23}{2^3 5^2}$ will have terminating decimal expansion.

(vii) $\frac{129}{2^2 5^7 7^5}$

Here, denominator $2^2 5^7 5^5$ is not of the form $2^2 5^7 7^5$ is not of the form $2^n 5^m$ exactly, so the given rational number $\frac{129}{2^2 5^7 7^5}$ will not have terminating decimal expansion. It will have a non-terminating repeating decimal expansion.

(viii) $\frac{6}{15}$

Here, denominator,

$$15 = 3 \times 5 = 3^1 5^1$$

Which is not of the form $2^n 5^m$, so the given rational number will have a non-terminating repeating decimal expansion.

(ix) $\frac{35}{50}$

Here $50 = 2 \times 5 \times 5 = 2^1 5^2$

Which is of the form $2^n 5^m$, so the given rational number $\frac{35}{50}$ will have terminating decimal expansion.

(x) $\frac{77}{210}$

Here, $210 = 2 \times 3 \times 5 \times 7$

$$= 2^1 \times 3^1 \times 5^1 \times 7^1,$$

Which is not of the form $2^n 5^m$, so the given rational number will have a non-terminating repeating decimal expansion.

PRACTICE :

1. Without actually performing the long division, state whether the following rational numbers will have *a* terminating decimal expansion or *a* non-terminating repeating decimal expansion.

(i) $\frac{786}{1500}$ (ii) $\frac{231}{36}$

(iii) $\frac{305}{108}$ (iv) $\frac{57}{625}$

(v) $\frac{31}{30}$ (vi) $\frac{121}{2^3 \times 3^2 \times 7^5}$

(vii) $\frac{32}{455}$ (viii) $\frac{3}{500}$

(ix) $\frac{17}{625}$ (x) $\frac{19}{3125}$

Ans : (i) T (ii) NTR (iii) NTR (iv) T (v) NTR (vi) NTR (vii) NTR (viii) NTR (ix) T (x) T

2. Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Sol :

(i) $\frac{13}{3125}$

We have, $\frac{13}{3125} = \frac{13}{5 \times 5 \times 5 \times 5 \times 5} = \frac{13}{5^5}$

Multiplying by 2^5 in numerator and denominator,

$$\frac{13}{3125} = \frac{13 \times 2^5}{5^5 \times 2^5} = \frac{13 \times 22}{10^5}$$

$$= \frac{416}{10^5} = 0.00416$$

(ii) $\frac{17}{8}$

We have, $\frac{17}{8} = \frac{17}{2^3}$

Multiplying by 5^3 in numerator and denominator,

$$\frac{17}{8} = \frac{17 \times 5^3}{2^3 \times 5^3} = \frac{17 \times 125}{10^3}$$

$$= \frac{2125}{1000} = 2.125$$

(iii) $\frac{64}{455}$

We have, $\frac{64}{455} = \frac{64}{5 \times 7 \times 13}$

Here, denominator is not of the form $2^n 5^m$, so the given rational number represents non-terminating repeating decimal expansion.

(iv) $\frac{15}{1600}$

We have, $\frac{15}{1600} = \frac{15}{2^6 \times 5^2}$

Multiplying by 5^4 in numerator and denominator,

$$\frac{15}{1600} = \frac{15}{2^6 \times 5^6} \times 5^4$$

$$= \frac{15 \times 625}{10^6}$$

$$= \frac{9375}{10^6} = 0.009375$$

(v) $\frac{29}{343}$

We have, $\frac{29}{343} = \frac{29}{7 \times 7 \times 7} = \frac{29}{7^3}$

Here, denominator of the given rational number is not of the form $2^n 5^m$, so the given rational number represents non-terminating repeating decimal expansion.

(vi) $\frac{23}{2^3 5^2}$

Multiplying by 5^1 in both numerator and denominator we have,

$$\frac{23}{2^3 5^2} = \frac{23 \times 5}{2^3 5^3} = \frac{115}{10^5} = 0.115$$

(vii) $\frac{129}{2^2 5^7 7^5}$

We have, $\frac{129}{2^2 \times 5^7 \times 7^5}$,

Here, denominator is not of the form $2^n 5^m$, so the given rational number represents non-terminating repeating decimal expansion.

(viii) $\frac{6}{15}$

We have, $\frac{6}{15} = \frac{2}{5} = \frac{2}{5^1}$

Multiplying by 2^1 in both numerator and denominator,

$$\frac{6}{15} = \frac{2 \times 2}{5^1 \times 2^1} = \frac{4}{10} = 0.4$$

(ix) $\frac{35}{50}$

We have, $\frac{35}{50} = \frac{7}{10} = \frac{7}{2^1 \times 5^1} = 0.7$

(x) $\frac{77}{210}$

We have, $\frac{77}{210} = \frac{11}{30} = \frac{11}{2 \times 3 \times 5}$

Here, 'denominator' is not of the form $2^n 5^m$, so the given rational number represents a non-terminating repeating decimal expansion.

3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational, or not. If they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factors of q?

(i) 43.123456789
(ii) 0.120 1200 12000 120000...
(iii) $43.\overline{123456789}$

Sol :

(i) 43.123456789

Since, the decimal expansion terminates, so the given real number is rational and therefore of the form $\frac{p}{q}$.

$$\frac{p}{q} = \frac{43123456789}{1000\,000\,000}$$

$$= \frac{431231456789}{10^9}$$

$$= \frac{43123456789}{(2 \times 5)^9}$$

$$= \frac{43123456789}{2^9 5^9}$$

Here, $q = 2^9 5^9$

The prime factorization of q is of the form $2^n 5^m$

$2^n 5^m$, where $n = 9$

$m = 9$

(ii) 0.120 1200 12000 120000 ....

Since, the decimal expansion is neither terminating nor non-terminating repeating, therefore, the given real number is not rational.

(iii) $43.\overline{123456789}$

Since, the decimal expansion is non-repeating, therefore, the given real number is rational and therefore of the form $\frac{p}{q}$.

Let, $x = 43.\overline{123456789}$

$x = 43.123456789$ ...(i)

Multiplying both sides of (i) by 100000000, we get

$100000000x = 43123456789.123456789$ ...(ii)

Subtracting (i) from (ii), we get

$9999999999x = 43123456746$

$$x = \frac{43123456746}{999999999}$$

$$= \frac{14374485582}{333333333}$$

Here, $q = 333333333$ which is not of the form $2^n 5^m$, n, $m \varepsilon I$.

PRACTICE :

1. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational, or not. If they are rational, and of the form $\frac{q}{q}$, what can you say about the prime factors of q?

(i) 5.567

(ii) 2.3010800012

(iii) $3.\overline{567}$

Ans : (i) Multiple of 2 or 5 only (ii) Not rational (iii) factors other than 2 or 5 also

2. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational, or not. If they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factors of q?

(i) 9.587587587......

(ii) 4.5321

(iii) 8.201318204213

Ans : (i) factors other than 2 or 5 also (ii) Multiple of 2 or 5 only (iii) Not rational

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